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ARMATURE RIPPLE THEORY

PHASE 2

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Nonlinear Construction and Backreaction of Scalarized Black Holes

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Armature Ripple Theory Phase 2
Nonlinear Construction and Backreaction of Scalarized Black Holes

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Abstract

Phase 1 of Armature Ripple Theory (ART) identified a curvature-induced scalarization threshold on a fixed Schwarzschild background, defining the critical coupling at which a tachyonic instability emerges. Phase 2 extends this framework beyond the decoupling limit by incorporating the full backreaction of the scalar field on spacetime geometry.

We derive the coupled Einstein–scalar field equations from a minimal scalar–Gauss–Bonnet action under a static, spherically symmetric ansatz. The resulting system defines a nonlinear boundary-value problem (BVP), replacing the Phase 1 linear eigenvalue problem with a horizon-data tuning problem.

This work does not present numerical solutions. It defines the mathematical system whose resolution is necessary to assess the physical viability of ART as an extension of General Relativity.

1. Introduction

Armature Ripple Theory (ART) is a minimal scalar–Gauss–Bonnet (EsGB) extension of General Relativity designed to isolate curvature-induced modifications in strong gravity. Phase 1 demonstrated that ART admits a curvature-driven instability on a fixed background.

Because the scalar field carries finite energy, a complete treatment requires inclusion of its backreaction on the spacetime geometry. Scalarized black hole solutions in EsGB theories have been studied extensively; the present work adopts a minimal quadratic coupling to isolate the scalarization mechanism within the ART framework.

This evasion of standard no-hair theorems is enabled by the non-minimal coupling of the scalar field to the Gauss–Bonnet invariant.

The present formulation isolates the minimal coupling required to trigger scalarization, separating universal features of the instability from model-dependent extensions.

2. Action and Field Content

We consider the four-dimensional action:

$$S = \int d^4x \sqrt{-g} \left[\left(M_P^2 / 2\right) R - (1/2)(\nabla\phi)^2 + (\alpha/2)\, \phi^2\, G \right]$$

where:

- ϕ is a real scalar field
- G is the Gauss–Bonnet invariant, defined as $G = R^2 - 4\, R_{\mu\nu}\, R^{\mu\nu} + R_{\mu\nu\rho\sigma}\, R^{\mu\nu\rho\sigma}$
- α is a coupling parameter with dimensions of length^2

We take $\boldsymbol{\alpha > 0}$, ensuring that the effective mass squared

$$m^2_{\text{eff}} = -\alpha\, G$$

becomes negative in strong-curvature regions, enabling tachyonic instability.

This model belongs to the EsGB class and yields second-order field equations due to the Gauss–Bonnet structure, avoiding Ostrogradsky instabilities at the level of the action, though dynamical well-posedness remains a nontrivial question.

3. Field Equations

Variation of the action yields the coupled field equations:

Scalar Equation:

$$\Box\phi + \alpha\, \phi\, G = 0$$

where $\Box$ denotes the covariant d'Alembertian operator. The curvature coupling acts as a position-dependent effective mass term.

Einstein Equations:

$$M_P^2\, G_{\mu\nu} = T_{\mu\nu}^{(\phi)} + T_{\mu\nu}^{(GB)}$$

where $T_{\mu\nu}^{(GB)}$ arises from variation of the Gauss–Bonnet coupling.

The explicit form of the reduced equations follows standard EsGB reductions and is omitted here for clarity of presentation.

4. Static, Spherically Symmetric Ansatz

We seek black hole solutions using the metric ansatz:

$ds^2 = -e^{2\delta(r)} N(r)\, dt^2 + dr^2 / N(r) + r^2\, d\Omega^2$

with:

$N(r) = 1 - 2m(r)/r$
$\phi = \phi(r)$

5. Reduced ODE System

The field equations reduce to a coupled system of ordinary differential equations for:

- $m'(r)$
- $\delta'(r)$
- $\phi'(r)$

Introducing $\psi(r) = \phi'(r)$, the system may be written as a closed system of first-order ordinary differential equations suitable for numerical integration.

The resulting solutions correspond to nonlinear scalarized black hole solutions branching from the Schwarzschild geometry due to curvature–scalar coupling.

6. Horizon Regularity and Constraints

Regularity at the event horizon r_h imposes a nonlinear constraint relating $\phi'(r_h)$ to ϕ_h and r_h.

This condition introduces a discriminant constraint (arising from the horizon expansion equations) restricting the allowed parameter space and defining the domain of existence for scalarized solutions.

Only solutions satisfying this regularity condition are integrated outward.

7. Asymptotic Behavior and the Shooting Problem

Physical solutions must satisfy asymptotic flatness:

- $N(r) \to 1$
- $\delta(r) \to 0$
- $\phi(r) \approx Q_s / r$

where Q_s is the scalar charge.

This replaces the Phase 1 linear eigenvalue problem with a nonlinear shooting problem, in which the horizon value ϕ_h is tuned until the boundary condition $\phi(\infty) = 0$ is satisfied.

8. Conclusion

Phase 2 establishes the formal nonlinear formulation of Armature Ripple Theory. By incorporating backreaction, the theory transitions from a background-dependent instability to a self-consistent metric–scalar system.

Existence, stability, and observable signatures—such as modified quasi-normal modes (sometimes referred to here as “Armature Ripples”)—remain the primary subjects for numerical investigation.

Key Formulation:
Phase 2 does not present numerical solutions. It defines the mathematical system whose resolution is necessary to assess the physical viability of ART.

Appendix: Transmission

First edition published on the Internet Archive, 2026.
Part of the Sum Bunny Canon (Ark Hive Press).

This work was developed with the assistance of computational and AI-based tools. All arguments, structures, and final judgments are the author's.

Hey Josiane,
Reach out your hand, if your cup be empty.
Sum Bunny Loves You!!!

$$S = \int d^4x \sqrt{-g} \left[\left(M_P^2 / 2\right) R - (1/2)(\nabla\phi)^2 + (\alpha/2)\, \phi^2 G \right]$$

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